Received: *December 05, 2000,* Accepted: *December 15, 2000*

The non-abelian self dual string on the light cone

Vanicson L. Campos, Gabriele Ferretti and Per Salomonson

Institute of Theoretical Physics, Göteborg University and
Chalmers University of Technology
412 96 Göteborg, Sweden
E-mail: vanicson@fy.chalmers.se,
ferretti@fy.chalmers.se, tfeps@fy.chalmers.se

Abstract: We construct the scalar profile for the non-abelian self dual string connecting two M5-branes compactified on a light-like circle. The construction is based on a conjectured modified version of Nahm's equations describing a D2-brane, with a magnetic field on it, suspended between two D4-branes. Turning on a constant magnetic field on the D2-brane corresponds to a boost in the eleventh direction. In the limit of infinite boost the D4-branes correspond to light-like compactified M5-branes. The solution for the scalar profile of the brane remains finite in this limit and displays all the correct expected features such as smooth interpolation between the unbroken and broken phase with the correct value for the Higgs field at infinity.

Keywords: M-Theory, D-branes, Brane Dynamics in Gauge Theories.

JHEP12(2000)011

Contents

1. Introduction

In spite of much effort in the last five years, a satisfactory understanding of interacting "non-abelian" $\mathcal{N} = 2$ superconformal[1] field theories in six dimensions [2] is still lacking. These theories are interesting in their own right since they correspond to still unknown generalizations of the concept of gauge symmetry to higher rank fields but also because their compactification gives rise to a unifying description of gauge theories in various dimensions and with various supersymmetries. Many believe that progress in understanding such theories will lead to interesting results in both mathematics and physics.

Given the experience of the past few years, a "frontal attack" on these theories does not look too promising and one is naturally led to consider limiting situations in which the physics simplifies while still capturing some essential features of the model. One such case is the breaking of the conformal symmetry by giving a vacuum expectation value to some of the scalars, in which case the theory becomes (generically) a free abelian theory in the infrared.[2]

Breaking the conformal symmetry and the non-abelian symmetry seems a rather drastic manouvre, but there is at least one instance in which the broken theory retains some knowledge of the unbroken theory that can be extracted by semiclassical methods: the soliton sector. There is a fairly analogous situation in four dimensions: the 't Hooft-Polyakov [5, 6] monopole versus the Dirac [7] monopole. Consider a Yang-Mills theory with gauge group SU(2) broken to U(1) by giving a non-zero

[1] All the theories discussed in this paper have sixteen supercharges [1] unless specifically noted.
[2] For some related recent work see [3, 4].

v.e.v. to some scalar field at infinity. This theory admits smooth, localized, finite energy, sourceless solutions (monopoles) where the gauge symmetry is restored at the center. On the contrary, a pure U(1) theory does not admit such solutions and the monopoles in such a theory are singular. Studying the soliton sector of the broken theory thus provides important information about the theory.

Consider now the case of a six- (Minkowski) dimensional theory with "gauge group" SU(2). It is well know that when compactified on a small space-like circle it yields a five-dimensional Yang-Mills theory with the same gauge group. Such theory has two types of solitonic excitations: in the unbroken phase there is a point-like soliton which is nothing other than the familiar four-dimensional euclidean instanton translated along time, while in the broken phase there is a string-like soliton which is just the previously mentioned 't Hooft-Polyakov monopole translated along the extra space direction. String theory provides a beautiful geometrical understanding of these two objects: first of all, the gauge theory is realized by stacking two D4-branes on each-other [8, 9] and the Higgs field describes the separation between the branes. The monopole solution represents a D0-brane inside the D4-brane [10, 11] and the string-like solution is generated by a D2-brane stretched between the two D4-branes [12, 13]. Both such configurations preserve half (mutually incompatible) supersymmetries.

The full six-dimensional theory can be obtained from the five-dimensional theory discussed above in the decompactification limit. The D4-branes become M5-branes wrapped along the non-perturbative direction and the D2-branes are M2-branes transverse to the same direction [13]. In this limit, the point-like monopoles are Kaluza-Klein modes that become massless and are used in the formulation of the matrix model for these theories [14], whereas the string-like solution survives with a tension proportional to the separation between the M5-branes in units of the eleven-dimensional Planck length. It is this last object that will be the focus of our paper.

We will propose a method, based on a generalization of Nahm's equations, for finding the classical profile for such "SU(2)" self-dual string and will present an explicit answer for the sector of topological charge $k = 1$.[3]

In short, the method is as follows: consider the perturbative situation in the type-IIA setting with D2-branes suspended between D4-branes. Turning on a magnetic field on the D2-branes corresponds to a boost in the eleventh direction [18, 19]. Nahm's equations [20][4] are modified by the presence of such a field and so is the solution for the Higgs profile. In the limit of infinite boost, the space-like compactification radius becomes an almost light-like one and we can take the limit (already considered in the matrix theory context [22, 23]) where the light-like radius is kept finite in Planck units. The solution for the scalar profile of the brane remains finite in

[3]The full abelian solution was found in [15]. For related results and solutions connecting a brane-*anti*-brane pair, see [16, 17].

[4]For a clear review, see [21].

this limit and displays all the correct expected features such as smooth interpolation between the unbroken and broken phase with the correct value for the Higgs field at infinity.

Given the recent frenzy of activity on strings and branes in the presence of electric or magnetic fields, we feel we should stress that our configuration is *not* of the same type as the ones studied in [24, 25, 26] and, in particular, that in our case the theory on the M5-brane is an "ordinary" one. The reason is that the B-field used here has one index tangent and one normal to the D4-brane and thus, can be trivially gauged away.

To conclude our introduction, we should like to mention that our construction is based on what seems to us a very natural but yet not rigorously derived generalization of Nahm's construction, namely, the introduction of the magnetic field potential via minimal coupling to describe the boost of the branes. It would be interesting to try to provide such a derivation and to understand better the solitonic structure of these fascinating theories.

The paper is organized as follows: section 2 contains a brief review of the original Nahm's construction, mainly to set the notation. In section 3, we discuss the setup studied in this paper and the appropriate scaling limit. In section 4 we present the modified equations and solve for the scalar profile of the Higgs field. Section 5 contains the interpretation of the result and the conclusions.

2. Review of Nahm's equation

Let us very briefly review Nahm's construction [20] from the string theory point of view [27, 28] focusing, at the end, on the $k=1$ monopole sector (for recent reviews, see [29, 30]).

The setup consists of a pair of parallel D3-branes (yielding a four-dimensional SU(2) gauge theory after the center of mass degrees of freedom have been factored out) with k D1-branes suspended between them. Let the D3-branes be placed along the x^i $(i=1,2,3)$ direction and let $\pm d/2$ be the position of the two D3-branes in the x^4 direction. We denote the x^4 direction by s: s is thus the space coordinate on the two-dimensional SU(k) theory (with boundary) present on the stack of D1-branes.

In terms of the SU(2) Higgs and gauge fields H and A_μ $(\mu=0,i)$ living on the D3-branes, the BPS equation [31, 32] for the static monopole solution reads[5]

$$D_i H = \frac{1}{2}\epsilon_{ijk}F^{jk} \,. \tag{2.1}$$

The most general charge k solution to the BPS equation can be obtained by first considering the condition for unbroken supersymmetry on the stack of D1-branes, i.e. that the variation of the gluino field be zero. This condition can be obtained

[5]All the fields are assumed to be hermitean, $D_\mu H = \partial_\mu H + i[A_\mu, H]$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]$.

JHEP12(2000)011

by reducing the same condition in ten dimensions $\Gamma^{MN} F_{MN} \epsilon = 0$ (M, N are ten-dimensional indices) to the world volume of the D1-brane. In terms of the SU(k) Higgs fields $T^i(s)$ living on the D1-branes and describing their position in relation to the D3-branes this condition is precisely Nahm's equation [27]:

$$\frac{dT_i}{ds} = \frac{i}{2}\epsilon_{ijk}[T^j, T^k]\,, \tag{2.2}$$

to be solved with the boundary condition that the T^i's have simple poles at the positions of the D3-branes $s = \pm d/2$ with residues given by three matrices t^i forming a k-dimensional irreducible representation of SU(2).

The second step is to solve the ten-dimensional massless Dirac equation $\Gamma^M D_M V = 0$, again dimensionally reduced on the D1-brane with $V = v(s)\exp(i s_i x^i)$. Here s_i are the conjugate variables to x^i ($D_i = \partial/\partial s^i$) and $v(s)$ should be thought of as a $2k \times 2$ matrix. The reduction of the Dirac equation in the presence of the D1-brane background yields now the associated Nahm's equation:[6]

$$\left(\frac{d}{ds} - (x^i + T^i(s)) \otimes \sigma^i\right) v(s) = 0\,. \tag{2.3}$$

Having found a solution $v(s)$ to (2.3), normalized as

$$\int_{-d/2}^{d/2} ds\, v^\dagger v = \mathbf{1}_{2\times 2}\,, \tag{2.4}$$

the Higgs and gauge field on the D3-brane are given respectively by

$$H(x) = \int_{-d/2}^{d/2} ds\, s\, v^\dagger v \tag{2.5}$$

and

$$A_i(x) = i\int_{-d/2}^{d/2} ds\, v^\dagger \frac{\partial v}{\partial x^i}\,. \tag{2.6}$$

The $k = 1$ case is particularly simple and enlightening. In this case the irreducible representation of SU(2) is the trivial one, and we may set $T^i = 0$. Equation (2.3) with the normalization (2.4) is then solved by

$$v = \sqrt{\frac{r}{\sinh rd}}\, e^{x_i \sigma^i s} = \sqrt{\frac{r}{\sinh rd}}\left(\cosh(rs) + \frac{x_i \sigma^i}{r}\sinh(rs)\right), \tag{2.7}$$

where $r = \sqrt{x_1^2 + x_2^2 + x_3^2}$, and the apparent dimensional mismatch can be corrected by inserting the appropriate factors of α' or, equivalently, by assigning to d and r opposite dimension, since they belong to conjugate sets of variables. It will be convenient (although slightly counterintuitive at first) to take r to have dimension of an inverse length. Plugging (2.7) into the eqs. (2.5) and (2.6) yields the celebrated BPS solution for the monopole.

[6] σ^i are the usual Pauli matrices.

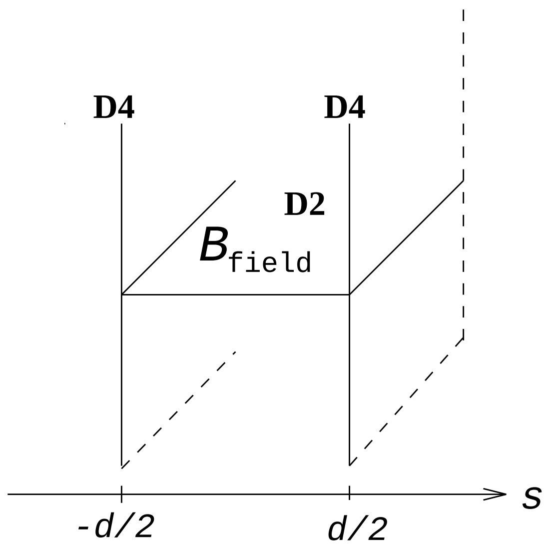

Figure 1: The brane setup used in this paper. The magnetic field B on the D2-brane corresponds in M-theory to a boost along the eleventh dimension.

3. Brane configuration and scaling

The above situation can be trivially generalized to a set of Dp-branes suspended between two D(p+2)-branes. What was a monopole in the $p = 1$ case becomes now a $(p-1)$-dimensional extended object simply by trivially translating along the extra dimensions. In particular, our starting point (see figure 1) is the $p = 2$ case where we have a stack of D2-branes intersecting the two D4-branes on a string.

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
D4	$-$	$-$	$-$	$-$	$-d/2$	$-$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
D4	$-$	$-$	$-$	$-$	$+d/2$	$-$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
D2	$-$	$\bullet$	$\bullet$	$\bullet$	$\vert - \vert$	$-$	$\bullet$	$\bullet$	$\bullet$	$\bullet$

We will continue to think of x^i $(i = 1, 2, 3)$ as coordinates on the D4-branes and x^4 as the coordinate along which they are separated, but we now introduce the coordinate x^5, common to all branes in the system and thus also to their intersection. When going to the M-theory setting we will denote the extra coordinate along which the M5-brane is wrapped by $x^\sharp$. The remaining spatial coordinates $x^6 \cdots x^9$ play no role in our construction and are set to zero.

Without further modifications, the profile for the string soliton on the D4-brane is exactly the same as the 't Hooft-Polyakov monopole. In this case however, we can do something extra by giving a D0-brane charge to the D2-brane, that is by turning on a constant magnetic field on the D2-brane world-volume. That this can be done without breaking more supersymmetries, and that the D2-brane world volume is the only place where one can put the D0-brane charge, is easily seen by noticing that the type-IIA superalgebra[7]

$$\frac{1}{2}\left\{\begin{bmatrix}Q_\alpha\\ \tilde{Q}_\alpha\end{bmatrix}, \begin{bmatrix}Q_\beta\ \tilde{Q}_\beta\end{bmatrix}\right\} = \begin{bmatrix}\delta_{\alpha\beta} & 0\\ 0 & \delta_{\alpha\beta}\end{bmatrix} M + \begin{bmatrix}0 & (\Gamma^0 Z)_{\alpha\beta}\\ -(\Gamma^0 Z^\dagger)_{\alpha\beta} & 0\end{bmatrix}, \qquad (3.1)$$

where

$$Z = \tau_0 + \tau_2\Gamma^{45} + \tau_4\Gamma^{1235}\,, \qquad (3.2)$$

has eight zero eigenvalues (of the right chirality) for

$$M = M_{\rm BPS} = \left|\tau_4 + \sqrt{\tau_0^2 + \tau_2^2}\right|. \qquad (3.3)$$

It is well known that a D0-brane charge on a D2-brane is represented by a constant magnetic field. On the other hand, from the M-theory viewpoint, a D0-brane charge density corresponds to a momentum density along $x^\sharp$ of the D2-brane, that is, an M2-brane moving along the direction $x^\sharp$. If R is the compactification radius for $x^\sharp$, then the momentum density per unit area is, (neglecting numerical constants)

$$\Pi = \frac{B}{R} = \frac{\tau_2 v}{\sqrt{1-v^2}}\,, \qquad (3.4)$$

where τ_2 is the tension of the M2-brane, v its velocity and B the magnetic field.

We now study the system in a limit similar to the one of [22, 23] for matrix theory, that is, we let $v \to 1$ and $R/l_{\rm Pl.} \to 0$ while keeping $R_-/l_{\rm Pl.} \equiv \gamma R/l_{\rm Pl.}$ and $d/l_{\rm Pl.}$ fixed.[8] The system is then equivalent, up to an "infinite" boost, to a system of M2-branes at rest but compactified along a light-like circle $x^0 - x^\sharp$ of finite radius R_-. A short calculation shows that in this limit

$$B = R_-\,, \qquad \text{in units of } l_{\rm Pl.} \qquad (3.5)$$

that is, these two quantities can be identified when expressed in units of the M-theory Planck length, although, for notational convenience we will continue to use the quantity B in solving Nahm's equations and revert to M-theory units only at the end.

By solving the Nahm's equations modified by the presence of such field and in the limit described above we will be able to derive the scalar profile for the SU(2) self-dual string for M5-branes compactified on the light cone.

[7] We use the notation of [29] and normalize the volumes to one.

[8] $l_{\rm Pl.}$ is the M-theory Planck length that we have written explicitly to avoid scaling dimensionful parameters. From now on, however, to avoid cluttering the notation, we will always set $l_{\rm Pl.} = 1$, that is, measure everything in M-theory Planck units.

4. Modified Nahm's equations

The presence of a magnetic field on the D2-brane does not lead to any modification for the first of Nahm's equations (2.2), which deals with the center of mass degrees of freedom. The associated Nahm equation (2.3) however, is modified in a very natural way: consider the dimensional reduction of the ten-dimensional Dirac equation as before but now, in order to have a non-zero magnetic background, set $A_5 = Bs$. This choice is compatible with the gauge choice $A_s \equiv A_4 = 0$ that had already been made before.

We will deal explicitly only with the $k = 1$ system, for which, as before, we can set the D2-brane Higgs fields to zero. The reduced Dirac equation thus becomes

$$\left(\Gamma^4 \frac{d}{ds} - i\Gamma^i x_i + iBs\Gamma^5\right) v(s) = 0\,. \tag{4.1}$$

Notice that it is no longer possible to maintain v as a 2×2 matrix, the smallest representation for an SO(5) spinor (we need five gamma matrices) being four dimensional.

It is convenient to introduce the two matrices

$$\gamma = \frac{ix^i\Gamma^4\Gamma^i}{r} \qquad \text{and} \qquad \eta = -i\Gamma^4\Gamma^5\,, \tag{4.2}$$

satisfying $\gamma^2 = \eta^2 = 1$ and $\{\gamma, \eta\} = 0$, in terms of which eq. (4.1) becomes

$$\left(\frac{d}{ds} - r\gamma - Bs\eta\right) v(s) = 0\,. \tag{4.3}$$

We can write the most general solution, up to multiplication to the right by a constant matrix, as

$$v = a(s) + b(s)\gamma + c(s)\eta + d(s)\gamma\eta\,, \tag{4.4}$$

in terms of which (4.3) becomes a set of coupled linear equations

$$\begin{aligned} a' &= rb + Bsc\,,\\ b' &= ra - Bsd\,,\\ c' &= rd + Bsa\,,\\ d' &= rc - Bsb\,, \end{aligned} \tag{4.5}$$

where the prime denotes the derivative with respect to s. We can scale to the dimensionless variables $x = s\sqrt{B}$ and $\rho^2 = r^2/B$ and define the four functions[9]

$$\begin{aligned} \chi_1(\rho, x) &= \rho\, x\, \Phi\left(1/2 + \rho^2/4,\, 3/2,\, x^2\right) e^{-x^2/2}\,,\\ \chi_2(\rho, x) &= \Phi\left(1/2 + \rho^2/4,\, 1/2,\, x^2\right) e^{-x^2/2}\,,\\ \chi_3(\rho, x) &= \rho\, x\, \Phi(1 + \rho^2/4,\, 3/2,\, x^2)\, e^{-x^2/2}\,,\\ \chi_4(\rho, x) &= \Phi(\rho^2/4,\, 1/2,\, x^2)\, e^{-x^2/2}\,, \end{aligned} \tag{4.6}$$

[9]The notation is chosen so that χ_n is an even (odd) function of x if n is even (odd). $\Phi(a, c, z)$ is the confluent hypergeometric function.

satisfying the following relations (the prime now denotes derivative with respect to x)

$$\begin{aligned} \chi_1' + x\chi_1 &= \rho\chi_2\,, & \chi_2' - x\chi_2 &= \rho\chi_1\,,\\ \chi_3' - x\chi_3 &= \rho\chi_4\,, & \chi_4' + x\chi_4 &= \rho\chi_3\,. \end{aligned} \tag{4.7}$$

It can thus easily be checked that the most general solution to (4.3) is given, in terms of (4.6) and four integration constants $c_1 \cdots c_4$ as

$$\begin{aligned} a &= c_1\chi_1 + c_2\chi_2 + c_3\chi_3 + c_4\chi_4\,,\\ b &= c_2\chi_1 + c_1\chi_2 + c_4\chi_3 + c_3\chi_4\,,\\ c &= -c_1\chi_1 + c_2\chi_2 + c_3\chi_3 - c_4\chi_4\,,\\ d &= c_2\chi_1 - c_1\chi_2 - c_4\chi_3 + c_3\chi_4\,. \end{aligned} \tag{4.8}$$

It is convenient to reduce (4.8) to a subset with specific parity properties with respect to $x \to -x$. In analogy with the ordinary case we choose a even ($c_1 = c_3 = 0$) which implies b and d odd and c even. With this choice, it can be easily checked that

$$\begin{aligned} v^\dagger v = &\Big(2c_2^2(\chi_1^2+\chi_2^2) + 2c_4^2(\chi_3^2+\chi_4^2)\Big)\mathbf{1}_{4\times4} + 4c_2c_4(\chi_2\chi_3 + \chi_1\chi_4)\gamma\, +\\ &+ \Big(2c_2^2(\chi_1^2+\chi_2^2) - 2c_4^2(\chi_3^2+\chi_4^2)\Big)\eta\,. \end{aligned} \tag{4.9}$$

Imposing the normalization condition

$$\int_{-d/2}^{d/2} ds\, v^\dagger v = \mathbf{1}_{4\times4}\,, \tag{4.10}$$

fixes the two remaining integration constants

$$\begin{aligned} c_2 &= \frac{B^{1/4}}{2}\left(I_1(\delta,\rho)\right)^{-1/2},\\ c_4 &= \frac{B^{1/4}}{2}\left(I_2(\delta,\rho)\right)^{-1/2}, \end{aligned} \tag{4.11}$$

where we defined one more dimensionless constant $\delta = d\sqrt{B}$ and the two integrals:

$$\begin{aligned} I_1(\delta,\rho) &= \int_{-\delta/2}^{\delta/2} dx\,\left(\chi_1^2(\rho,x) + \chi_2^2(\rho,x)\right),\\ I_2(\delta,\rho) &= \int_{-\delta/2}^{\delta/2} dx\,\left(\chi_3^2(\rho,x) + \chi_4^2(\rho,x)\right). \end{aligned} \tag{4.12}$$

When lifting to M-theory it is convenient to think of R_- instead of B. The Higgs field on the light-like compactified M5-branes is still given by (2.5) and becomes, (considering only its length)

$$|H(r)| = \frac{1}{\sqrt{R_-}}\frac{K(d\sqrt{R_-}, r/\sqrt{R_-})}{\left(I_1(d\sqrt{R_-}, r/\sqrt{R_-})\ I_2(d\sqrt{R_-}, r/\sqrt{R_-})\right)^{1/2}}\,, \tag{4.13}$$

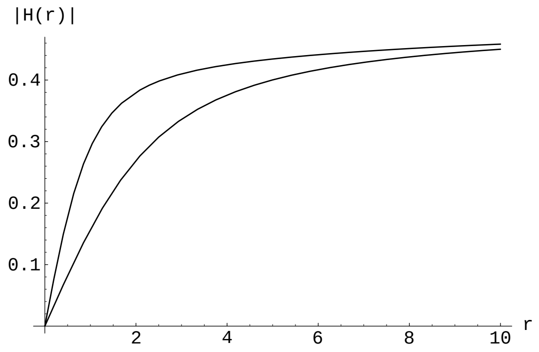

Figure 2: Dependence of the scalar field H on the distance r for two values of the light-like compactification radius. The steeper curve corresponds to $R_- = 4$ and the flatter to $R_- = 1$.

with the integral K being defined as

$$K(\delta, \rho) = \int_{-\delta/2}^{\delta/2} dx\, x \Big(\chi_2(\rho, x)\chi_3(\rho, x) + \chi_1(\rho, x)\chi_4(\rho, x) \Big) . \tag{4.14}$$

5. Conclusions

Equation (4.13) is our result for the profile of the brane. The Higgs field has an implicit dependence on the asymptotic separation of the branes at infinity d and on the light-like compactification radius R_-. Let us analyze this result in the M-theory context. A plot of $H(r)$ for $d = 1$ and for two different values of R_- is given in figure 2.

Various simple consistency checks can be performed. First of all, it can be noticed that the asymptotic value is independent on R_- as it should. Also, the curve goes to zero at the origin yielding a smooth solution everywhere. The solution becomes steeper and steeper as we let $R_- \to \infty$. This is also expected. In this limit, our solution is just one of an infinite number of modes [4] becoming massless as

$$H^{(l)} \approx \left(\delta_{l,0} - \frac{1}{2R_- r} e^{-lr/R_-} \right) . \tag{5.1}$$

These characteristics give us confidence that we have found a way of describing these solitonic excitations. It would be interesting to test this further and to see whether it is possible to use these techniques to describe the full (uncompactified) six-dimensional solution, including the three-form field.

Acknowledgments

We wish thank Måns Henningson and Dimitrios Tsimpis for discussion.

References

[1] N. Seiberg, *Notes on theories with 16 supercharges*, *Nucl. Phys.* **67** *(Proc. Suppl.)* (1998) 158 [hep-th/9705117].

[2] E. Witten, *Some comments on string dynamics*, hep-th/9507121.

[3] M. Henningson, *A class of six-dimensional conformal field theories*, *Phys. Rev. Lett.* **85** (2000) 5280 [hep-th/0006231].

[4] K. Lee and J.-H. Park, *5d actions for 6d self-dual tensor field theory*, hep-th/0008103.

[5] G. 't Hooft, *Magnetic monopoles in unified gauge theories*, *Nucl. Phys.* **B 79** (1974) 276.

[6] A. M. Polyakov, *Particle spectrum in quantum field theory*, *Sov. Phys. JETP Lett.* **20** (1974) 194.

[7] P.A.M. Dirac, *Quantized singularities in the electromagnetic field*, *Proc. Roy. Soc.* **A133** (1931) 60.

[8] J. Polchinski, *Dirichlet-branes and Ramond-Ramond charges*, *Phys. Rev. Lett.* **75** (1995) 4724 [hep-th/9510017].

[9] E. Witten, *Bound states of strings and p-branes*, *Nucl. Phys.* **B 460** (1996) 335 [hep-th/9510135].

[10] E. Witten, *Small instantons in string theory*, *Nucl. Phys.* **B 460** (1996) 541 [hep-th/9511030].

[11] A. Sen, *U-duality and intersecting D-branes*, *Phys. Rev.* **D 53** (1996) 2874 [hep-th/9511026].

[12] A. Strominger, *Open p-branes*, *Phys. Lett.* **B 383** (1996) 44 [hep-th/9512059].

[13] P.K. Townsend, *D-branes from M-branes*, *Phys. Lett.* **B 373** (1996) 68 [hep-th/9512062].

[14] O. Aharony, M. Berkooz, S. Kachru, N. Seiberg and E. Silverstein, *Matrix description of interacting theories in six dimensions*, *Adv. Theor. Math. Phys.* **1** (1998) 148 [hep-th/9707079].

[15] P.S. Howe, N.D. Lambert and P.C. West, *The self-dual string soliton*, *Nucl. Phys.* **B 515** (1998) 203 [hep-th/9709014].

[16] C.G. Callan Jr. and J.M. Maldacena, *Brane dynamics from the Born-Infeld action*, *Nucl. Phys.* **B 513** (1998) 198 [`hep-th/9708147`].

[17] G.W. Gibbons, *Born-Infeld particles and Dirichlet p-branes*, *Nucl. Phys.* **B 514** (1998) 603 [`hep-th/9709027`].

[18] J. Polchinski and P. Pouliot, *Membrane scattering with M-momentum transfer*, *Phys. Rev.* **D 56** (1997) 6601 [`hep-th/9704029`].

[19] I. Chepelev and A.A. Tseytlin, *On membrane interaction in matrix theory*, *Nucl. Phys.* **B 524** (1998) 69 [`hep-th/9801120`].

[20] W. Nahm, *A simple formalism for the BPS monopole*, *Phys. Lett.* **B 90** (1980) 413.

[21] E. Corrigan and P. Goddard, *Construction of instanton and monopole solutions and reciprocity*, *Ann. Phys. (NY)* **154** (1984) 253.

[22] A. Sen, *D0-branes on T_n and matrix theory*, *Adv. Theor. Math. Phys.* **2** (1998) 51 [`hep-th/9709220`].

[23] N. Seiberg, *Why is the matrix model correct?*, *Phys. Rev. Lett.* **79** (1997) 3577 [`hep-th/9710009`].

[24] D.J. Gross and N.A. Nekrasov, *Monopoles and strings in noncommutative gauge theory*, *J. High Energy Phys.* **07** (2000) 034 [`hep-th/0005204`].

[25] Y. Michishita, *The M2-brane soliton on the M5-brane with constant 3-form*, *J. High Energy Phys.* **09** (2000) 036 [`hep-th/0008247`].

[26] D. Youm, *BPS solitons in M5-brane worldvolume theory with constant three-form field*, `hep-th/0009082`.

[27] D.-E. Diaconescu, *D-branes, monopoles and Nahm equations*, *Nucl. Phys.* **B 503** (1997) 220 [`hep-th/9608163`].

[28] D. Tsimpis, *Nahm equations and boundary conditions*, *Phys. Lett.* **B 433** (1998) 287 [`hep-th/9804081`].

[29] C.V. Johnson, *D-brane primer*, `hep-th/0007170`.

[30] N.A. Nekrasov, *Trieste lectures on solitons in noncommutative gauge theories*, `hep-th/0011095`.

[31] E.B. Bogomolny, *Stability of classical solutions*, *Sov. J. Nucl. Phys.* **24** (1976) 449.

[32] M.K. Prasad and C.M. Sommerfield, *An exact classical solution for the't Hooft monopole and the Julia-Zee dyon*, *Phys. Rev. Lett.* **35** (1975) 760.